Theoretical modeling of the dependence between micro-contact parameters and mechanical properties of contact materials

Marilena Glovnea[1], Cornel Suciu [1), 2)], Ioan-Cozmin Manolache-Rusu[1]

[1)] Department of Mechanics and Technologies, Stefan cel Mare University of Suceava, Romania, 13 University Street, 720229, Stefan cel Mare University, Romania, e-mail: suciu@usm.ro

[2)] Faculty of Faculty of Electrical Engineering and Computer Science, Stefan cel Mare University, 720229 Suceava, Romania, e-mail: camil.suciu@student.usv.ro

As is well known, the solution of analytical elastic contact problems [1, 2] is limited to a small number of specific contacts. For this reason, various numerical approaches were proposed to solve contact problems. The use of conventional numerical solving methods is however limited by the large volume of computation due to the use of fine meshing of the assumed area of contact with an important number of nodes. Various methods are proposed in literature to reduce the computing time and power needed to solve contact problems.

In order to highlight the dependence between micro-contact parameters and the mechanical properties of the materials in contact, the present paper proposes the numerical modeling of micro-contacts using the Conjugate Gradient and Fast Fourier Transform technique (CG + FFT), [3, 4], and its adaptation to the micro-contact range. Model validation is performed by comparing the obtained results with the classical Hertz model, for various contact body shapes. The obtained results show very good agreement between the numerical and analytical results, thus validating the proposed numerical model.

Keywords: micro-contacts, numerical model, contact parameters, contact mechanics

1. INTRODUCTION

Accurate analytical solution of contact problems is limited to a small number of specific situations. As a result, in most situations, solving these problems by numerical methods is preferred. In order to simplify the problem, only the case of normal pressure (tensile) loading on the contact area will be further considered in this work. In this case, the contact area configuration and dimensions, the pressure distribution over this area and the normal approach of the contact bodies, result from numerically solving the equilibrium equation and the integral deformation conditions, while imposing satisfaction of the loading conditions outside the contact area and the regularity conditions towards infinity.

Over time, several numerical procedures have been proposed to solve elastic contact problems. These can be grouped as follows:

- direct integration or matrix inversion methods;
- finite difference methods;
- finite element methods;
- boundary element methods;
- point matching method;
- partial methods;
- fast numerical methods.

A comparison between different numerical methods was carried out by Gatina, [5], and by Johnson, [2]. In order to establish the merits of the various numerical methods, Gatina solved the classical problem in the case of three punch shapes, (paraboloidal, quasi-paraboloidal and circular conical), pressed against an elastic half-space. He used a direct integration method with a functional regularization, Kalker's method [7, 8], four finite difference methods and two finite element methods, (Laplace and Hermite). Of interest is the paraboloidal punch which yields the Hertz solution for pressure distribution and deformations.

The best results for maximum pressure were obtained by Kalker's method, (0.19% error), next is the RF method, (2.67% error for 9 cells and 1.94% for 33 cells). The finite element method led to the highest errors, 5% to 10%, while the finite difference method occupies an intermediate position, with errors between 0.57% and 5.5%. The best prediction of the normal approach, were obtained using the RF method and the Kalker method, both yielding the same error of 0.4%; at the opposite pole is the finite element method (error between 2.8% and 6.9%).

The best estimate of the contact area size is given by the RF method (error 2.2% for 9 cells and 0.7% for 33 cells), while the coarsest is given by the finite difference method, (errors of 4.82% to 12.15%). The second and third positions are occupied, in order, by the finite element method and Kalker's method, with errors of 2.73% and 3.77% respectively.

The shortest calculation time, 1284 seconds, is achieved by the RF method, 9 cells, while the longest, 33415 seconds, is achieved by the same method, but for 33 cells. The Kalker method requires between 1450 and 5800 seconds, the finite element method needs 3745 and 4045 seconds, while the best of the finite difference methods records 8382 to 16500 seconds.

A comparison of the results obtained in [5] for the other types of punches shows large differences between the shapes of pressure distributions, especially in the case of the conical punch, which shows surface singularities.

Johnson, [2], defined problems of concentrated contact, when the contact area is small compared to the extent of the bodies, and extended contact, where the contact stress field is an appreciable part of the overall stress field of the bodies. In the first case, the finite element method should be avoided, being suitable only for the second category of problems. In such circumstances, the boundary element method should also be considered.

In the present work, in order to highlight the dependence between micro-contact parameters and mechanical properties of contact materials, numerical modeling of the contact problem using the Conjugate Gradient and Fast Fourier Transform (CG+FFT) technique is proposed in [4]. Its validation on micro-contacts is performed by comparing the obtained results with the classical Hertz model.

2. THEORETICAL MODELING

It is well known that solving elastic contact problems analytically is limited to a small number of specific contact types. For this reason, a numerical approach is often employed for solving contact problems. The use of conventional numerical solution methods is limited by the large computational volume, due to the use of fine meshing of the assumed contact area, which yields a large number of nodes, all calculations being done in all those points.

In order to reduce computation times, various fast numerical techniques were developed over time, such as Multi Level Multi Summation (MLMS) and Fast Fourier Transform (FFT), [3, 9]. The two methods were used by Polonsky and Keer, [6], on a concentrated contact problem. The authors conclude that, in order to obtain accuracies comparable to that of the MLMS algorithm, in the case of applying the FFT algorithm it is necessary to extend the domain, with a negative effect on the required computational effort. The FFT technique applied to non-periodic problems introduces a periodicity error. Crețu, [8], developed a fast algorithm for the purpose of solving the real contact domain and pressure distribution for non-Hertz concentrated contacts.

The associated mathematical model is defined by:

1. The geometrical equation of elastic contact;
2. The integral equation of the elastic contact;
3. The equilibrium equation;
4. Conditions of smoothing and non-penetration.

In defining the mathematical model the following notations were used:

Q – normal load;

$p(x',y')$ – normal pressure distribution;

$z(x,y)=z_1(x,y)+z_2(x,y)$ – local contact geometry (initial distance between the surfaces in contact, in the absence of load);

A_c –real contact area;

η_s s – contact rigidity;

$w(x,y)=w_1(x,y)+w_2(x,y)$ – displacement along *z* axis;

δ_0 normal approach between contact bodies;

$r(x,y)$ – distance between surfaces after deformation.

The contact problem is solved numerically on a uniform rectangular grid of $M\times N$ dimension, included in the contact plane. A Cartesian coordinate system is introduced with the origin at the centre of symmetry of the grid, the *x* and *y* axes parallel to the sides of the grid, and the z axis normal to the tangent plane common to the surfaces in contact. The initial dimensions of the equivalent contact surface, the corresponding displacements in the Oz direction and the contact pressures, calculated at the nodes (i,j), are denoted by z_{ij} , w_{ij} and p_{ij} , and the grid cell sizes by *a* and *b*.

As described in [2], the equations shown below, represent the discrete formulation of the elastic contact problem:

$$r_{ij}=w_{ij}+z_{ij}-\delta_0\,,\ (i,j)\in A_c\ ; \tag{1}$$

$$w_{ij}=\sum_{k=1}^{M}\sum_{l=1}^{N} f_{|i-k|+1,|j-l|+1}\cdot p_{kl},\ 1\le i\le M,\ 1\le j\le N\ ; \tag{2}$$

$$Q=ab\cdot\sum_{i=1}^{M}\sum_{j=1}^{N}p_{ij}\ ; \tag{3}$$

$$r_{ij}=0,\ p_{ij}>0,\ (i,j)\in A_c\ ; \tag{4}$$

$$r_{ij}>0,\ p_{ij}=0,\ (i,j)\notin A_c\ . \tag{5}$$

In equation (2), $f_{|i-k|+1,|j-l|+1}$ are the influence coefficients that represent the displacement of the (i,j) node along *z* axis, due to unit pressure acting on element (k,ℓ).

Solving the problem of elastic contact by the Conjugate Gradient and Fast Fourier Transform method involves completion of several steps, as described further:

- Specifying the initial data of the elastic contact problem: the type of contact surfaces, the estimated contact area, the mesh dimensions along two perpendicular axial directions, the geometric elements of the contact surfaces, the elastic constants of the component materials, the normal force;
- Specifying a validation solution: in order to validate the algorithm and the related computer code, the elements of the elastic contact determined by various approaches (analytical, numerical or experimental) from the literature are stored;
- Discretization of the estimated contact area: a uniform rectangular network is generated, with M columns and N lines, included in the contact plane;
- Initial contact geometry: the elevations of the surfaces are calculated at the points that represent the centers of gravity of the discretization cells; the nominal geometry of the bodies in contact is taken into account;
- Calculation of influence coefficients.

The proposed mathematical model was applied to some classical problems of elastic contact, exemplifying below the elastic micro-contact between a paraboloidal punch and a flat elastic half-space.

Paraboloid – elastic half-space contact

The theoretical determination of the elements of an elastic ellipsoid – half-space contact was done using the method of influence coefficients, [3]. The estimated contact domain, which includes the actual contact area of unspecified shape and size, is a rectangular shaped domain discretized into a number of elementary areas called cells. On each cell the pressure is considered uniformly distributed. The contact stress is a purely normal load. The attached coordinate system originates from the domain's center of symmetry.

A variable-step discretization model is proposed, in which the axial dimensions of the cells are in decreasing arithmetic progression towards the area with large pressure gradients. The variable-step discretization is automatically generated after increasing the initial contact cell dimensions by multiplication factors.

Based on the discretization performed, a number of identifiers are associated to each cell such as: number, dimensions (length, width), vertex and center coordinates relative to the attached coordinate system.

Under the assumption of a uniformly distributed pressure on each elementary area of the estimated contact domain and based on the principle of superposition of effects, the numerical model was used with the equilibrium equation included in the system, a model defined by:

$$\begin{cases}\sum_{j=1}^{M\cdot N}(f_{i_0 j}-f_{ij})\cdot p_j=z_i\,,\ i=1....M\cdot N;\ i\ne i_0;\\ \sum_{j=1}^{M\cdot N}a_j\cdot b_j\cdot p_j=Q;\\ p_j>0, j\in I_c;\\ p_j=0, j\notin I_c,\end{cases} \tag{6}$$

where: f_{ij} are the coefficients of influence, signifying the displacement along normal direction of the cell centre (i) due to a unit pressure acting on cell (j);

a_j, b_j represent the dimensions of cell (j);

I_c is a set of indices highlighting the elementary component domains of the actual contact area;

p_j is the pressure on cell j;

M and N are the number of cells in axial directions (odd numbers).

3. RESULTS

In order to verify the proposed model, two simulations were conducted. For the first case, main contact parameters were determined both numerically and analytically for the contact between a spherical punch and an elastic half-space. The second considered case was that of a parabolic surface pressed against the same flat, elastic half-space. The considered parameters and results for the two situations are further presented.

Spherical punch - flat elastic half-space micro-contact

For the first presented case, an elastic micro-contact was considered between a rigid ($E_1=\infty$) spherical punch ($R_1=0.003\ m$), normally pressed against a synthetic glass flat surface, considered as a half-space with a Young's Modulus of $E_{Plexiglass}$=2380 MPa, and a Poisson's ratio of $\nu_{Plexiglass}$=0.35. The applied force was considered to be Q=200N. The potential contact domain was divided into 16384 square cells, leading to a 128×128 grid. For the described contact, both the proposed numerical model and the analytical Hertz model were applied, and the following contact parameters were determined: contact area, contact area radius, normal approach between the contact bodies (normal displacement) and maximum contact pressure.

Figure 1 graphically illustrates the 3D pressure distribution over the contact area, obtained using the proposed numerical model (Figure 1 a) and by use of the analytical model (Figure 1 b)

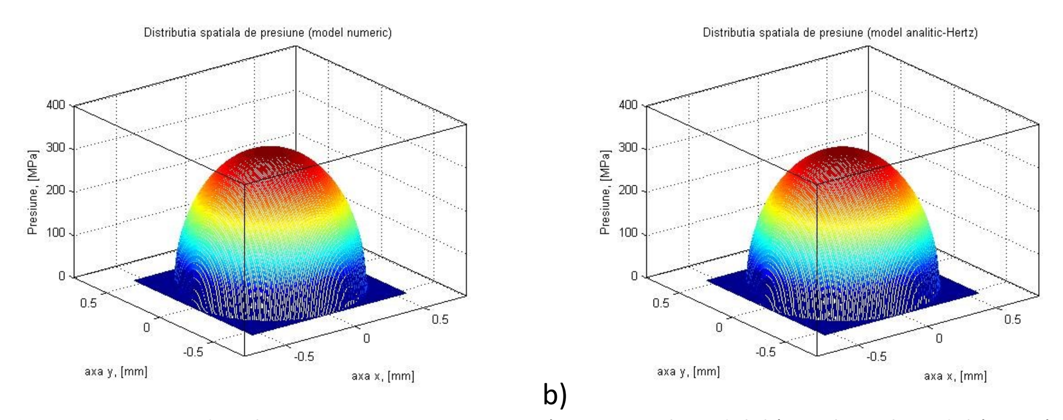

Figure 1 Pressure distribution over contact area: a) Numerical model, b) Analytical model (Hertz)

Figure 2 shows, on the same plot, the pressure distribution along the contact area radius, obtained analytically and numerically.

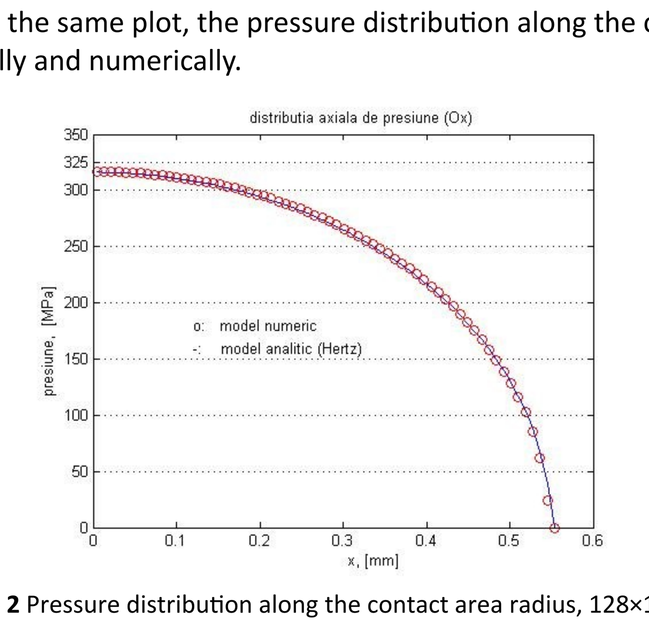

Figure 2 Pressure distribution along the contact area radius, 128×128 grid

The numerical values of significant contact parameters are presented in Table 1.

Table 1. Contact parameters obtained numerically and analytically for sphere – half-space contact

Method	Maximum pressure, [MPa]	Normal approach (penetration), [mm]	Contact area [mm^2]	Contact area radius [mm]
Analytic (Hertz)	316.806	0.1006	0.9473	0.5491
Numerical	316.244	0.1062	0.9496	0.5495

Parabolic roller – flat elastic half-space micro-contact

The Hertz elastic micro-contact between a rigid punch $(E_1=\infty)$, in the shape of a barrel roller with curvature radii $R_1=4.3\cdot10^{-3}\ m$ and $R_2=2.68\cdot10^{-3}\ m$, pressed normally on a Plexiglas half-space with the following constants was considered: $E_{Plexiglas}=2380\ MPa$, $\nu_{Plexiglas}=0.35$. The pressing force is $Q=200\ N$. The potential contact field has been divided into 529 square cells (23x23).

The curvature radii, principal curvatures at the initial point of contact and elastic parameters of the materials are given by equations (7):

$$\begin{gathered}R_1=4.3\cdot10^{-3}\,m\ ;\quad R_2=2.68\cdot10^{-3}\,m\ ;\\ k_{1x}=\frac{1}{R_1};\ k_{1y}=\frac{1}{R_1};\ k_{2x}=0;\ k_{2y}=0\ ;\\ \nu_2=0.35\ ;\quad E_1=\infty\ ;\quad E_2=2.38\cdot10^{11}\ Pa\ .\end{gathered} \tag{7}$$

Using the proposed mathematical model, the following elements were determined numerically and analytically:

- contact area;
- contact ellipse half-axes;
- normal approach between contact bodies;
- maximum contact pressure.

Figures 3-6 illustrate graphical representations for the geometry of the bodies in contact, the pressure distribution (analytically and numerically obtained) and the axial contact pressure distributions.

The numerical values of significant contact parameters are presented in Table 2.

Table 2. Contact parameters obtained numerically and analytically for parabolloid – half-space contact

Method	Maximum pressure [MPa]	Normal approach (penetration), [mm]	Contact area [mm^2]	Large half-axis (a) [mm]	Small half-axis (b) [mm]
Analitic	146.04	0.06252	2.054	1.488	0.439
Numeric	146.08	0.06248	2.10	1.488	0.418

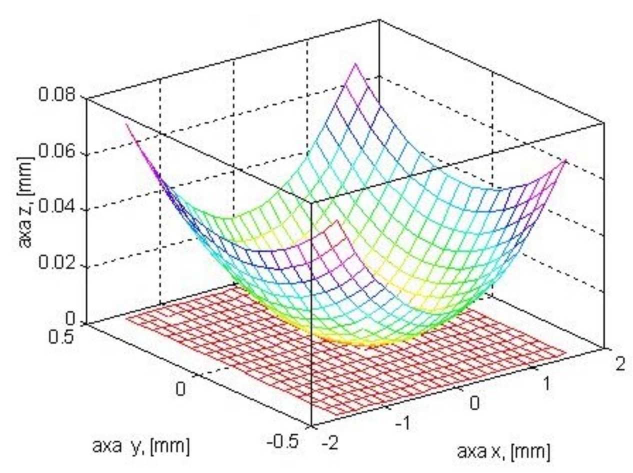

Figure 3. Geometry of boundary surfaces of the contact bodies (barrel roller – flat surface), (23x23)

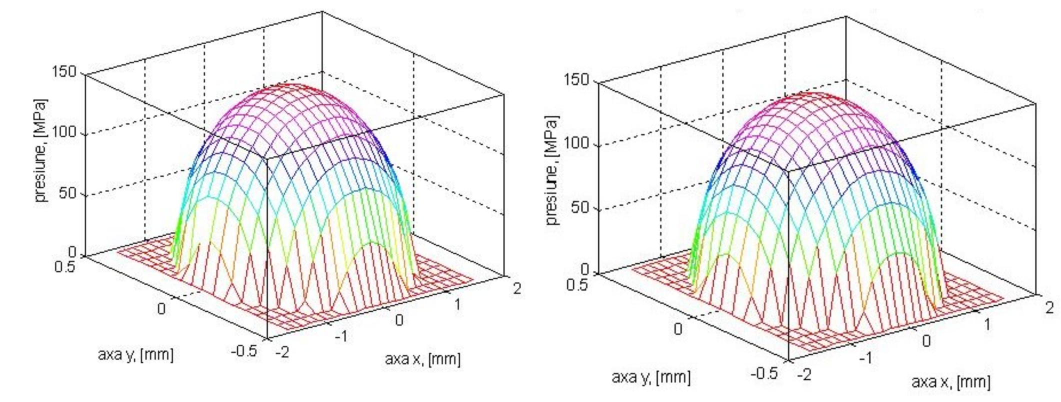

Figure 4. Spatial distribution of contact pressure in a barrel roller – plane contact: a) numerical model, (23x23); b) analytic model (23x23)

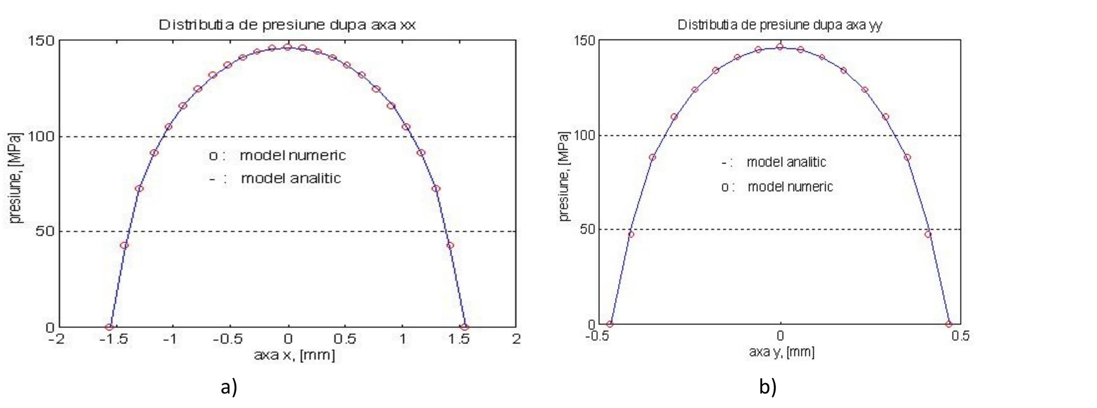

Figure 5. Axial contact pressure distributions, roller - plane contact, (23x23) a) - along the x-axis, b) - along the y-axis,

In the case of the roll-elastic half-space contact, a very fine discretization was not achieved, as in the case of the circular contact above. However, the graphs and numerical values show that the proposed mathematical model agrees very well with the results obtained with the Hertz theory.

4. CONCLUSIONS

In order to highlight the dependence between micro-contact parameters and the mechanical properties of the materials in contact, the present paper proposed a numerical modeling of micro-contacts using the Conjugate Gradient and Fast Fourier Transform technique (CG + FFT) and its adaptation to the micro-contact range. Model validation was performed by comparing the obtained results with the classical Hertz model, for two contact body shapes.

The proposed mathematical models used as parameters of the micro-contact, the contact area dimensions, the normal displacement of the contact bodies and the maximum contact pressure.

The elastic characteristics of the tested material were materialized by the longitudinal modulus of elasticity and the Poisson's ratio.

The obtained results show very good agreement between the numerical and analytical results, thus validating the proposed numerical model.

REFERENCES

1) Johnson, K.L., Contact Mechanics, Cambridge University Press, 1985.
2) Johnson, K.L., „Classical Versus Numerical Methods of Elastic Contact Stress Analysis", Proc. of Meet. Stress Analysis and Trib. Groups of Inst. Phys., 1-10, (1990).
3) Grădinaru, D., Modelări numerice în teoria contactului elastic, PhD Thesis (in Romanian), Stefan cel Mare University of Suceava, Romania, (2006);
4) Grădinaru, D., Modelări numerice în teoria contactului elastic, Ed. Did. Ped., ISBN 978-973-30-1863-6, (2007).
5) Gatina, J.C., Contacts des corps elastiques - Effets tangentiels et normaux. Formulation et resolution des problems inverse et direct, Thčse de Docteur d'Etat Es Sciences, INSA de Lyon-Lyon I, 1987.
6) Polonsky, I. A. and Leon M. Keer. "Fast Methods for Solving Rough Contact Problems: A Comparative Study." Journal of Tribology-transactions of The Asme 122 (2000): 36-41.
7) [Ka71] Kalker, J.J., A Minimum Principle for the Law of Dry Friction, with Application to Elastic Cylinders in Rolling Contact, Part 1, 2, ASME, Journal of Applied Mechanics, vol. 38, 4, 1971, 875-880; 881-887.
8) [Ka72] Kalker, J.J., Y. Van Randen, A Minimum Principle for Frictionless Elastic Contact with Applications to Non-Hertzian Half-Space Contact Problems, Journal of Engrg. Math., vol. 6, 2, 1972, 193-206.
9) Wang, Q.J., Sun, L.L., Zhang, X., Liu, S., & Zhu, D. (2020). FFT-Based Methods for Computational Contact Mechanics. Frontiers in Mechanical Engineering.