Fractals, the missing link between certainty and uncertainty

Catalin Silviu Nutu [1)]

[1)]*Department for General Engineering Sciences, Constanta Maritime University, Constanta, Romania*

"everything that has shape can be defined, everything that can be defined can be conquered"
Sun Bin

CONTENT

ANALYSIS OF FRACTALS' DYNAMICS FOR FRACTALS' SHAPES WITH UNCORRELATED COORDINATES

For a fractal's shape dynamic such as described in the present chapter of the paper, where the coordinates are not depending on each other, but only on themselves, the process can be written in system form:

$$\begin{cases} x_{i+1} = ax_i + c_i \\ y_{i+1} = by_i + d_i \end{cases}$$

And hence the corresponding matrix form is:

$$\begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix}(a, b) + \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$

The analysis of these dynamic fractals can be made twofold, in the time domain and in the frequency (spectral) domain. In order to analyze the above series in the time domain, one has to calculate:

a) the means of the series x, $E(x_i) = \mu_x$ and y, $E(y_i) = \mu_y$,
b) the simple covariance functions: $cov(x_t, x_{t+k}) = \gamma_{xx}(k)$ and $cov(y_t, y_{t+k}) = \gamma_{yy}(k)$
c) the autocorrelation functions or coefficients: $\rho_{xx}(k)$ and $\rho_{yy}(k)$

This analysis of both series is going to be made by calculating the autocovariance functions of both time series at one certain lag k: $\gamma(k)$ and $\Delta(k)$, and then to calculate out of them the (power) spectral density functions of both series: $f(\omega)$ and $g(\omega)$, based on formulas below:

$$\begin{cases} f(\omega) = \dfrac{1}{\pi}\sum_{-\infty}^{\infty} \gamma(k)e^{-i\omega k} \\ g(\omega) = \dfrac{1}{\pi}\sum_{-\infty}^{\infty} \Delta(k)e^{-i\omega k} \end{cases}$$

ANALYSIS OF FRACTALS' DYNAMICS FOR FRACTALS' SHAPES WITH CORRELATED COORDINATES

For a fractal's shape dynamic such as described in the present chapter of the paper, where the coordinates are depending not only on themselves but also on each other, but only on itself, the process can be written in system form:

$$\begin{cases} x_{i+1} = ax_i + by_i + e_i \\ y_{i+1} = cy_i + dy_i + f_i \end{cases}$$

which is actually an iterated function system that can be written in the following matrix form:

$$\begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_i \\ y_i \end{pmatrix} + \begin{pmatrix} e_i \\ f_i \end{pmatrix}$$

The analysis of this type of dynamic fractals can be made again, twofold, in the time domain and in the frequency (spectral) domain. In order to analyze the above series in the time domain, one has to calculate not only the same functions as for the fractals' shapes with uncorrelated coordinates, but also the cross covariance and cross correlation functions between the two interdependent series:

a) the means of the series x, $E(x_i) = \mu_x$ and y, $E(y_i) = \mu_y$,
b) the simple covariance functions: $cov(x_t, x_{t+k}) = \gamma_{xx}(k)$ and $cov(y_t, y_{t+k}) = \gamma_{yy}(k)$
c) the autocorrelation functions or coefficients: $\rho_{xx}(k)$ and $\gamma_{yy}(k)$
d) the cross covariance functions: $cov(x_t, y_{t+k}) = \gamma_{xy}(k)$, $cov(x_{t+k}, y_t) = \gamma_{xy}(-k)$, $cov(x_t, y_{t-k}) = \gamma_{xy}(k)$ and $cov(x_{t-k}, y) = \gamma_{xy}(-k)$, which are fulfilling the condition: $\gamma_{xy}(k) = \gamma_{xy}(-k)$
e) the cross correlation functions obtained by standardization, out of the cross covariance functions: $\rho_{xy}(k)$ and $\rho_{yx}(k)$, which are fulfilling the conditions: $\rho_{xy}(k) = \rho_{yx}(-k)$, and $-1 \leq \rho_{xy}(k) \leq 1$, where $\rho_{xy}(k)$ is given by the formula:

$$\rho_{xy}(k) = \frac{\gamma_{xy}(k)}{\sqrt{\gamma_{xx}(0)\gamma_{yy}(0)}}$$

This later cross correlation function with the above listed properties, measures the correlation between the observations x_t and y_{t+k} and is the proper tool to analyze the correlations between the two coordinates of the shape S(x, y) in the time domain. This presented analysis in the time domain can be easily extended to the continuous case scenario, where the series x and y are continuous functions of the variable time t, using the corresponding formulas.

This analysis of the above process in the spectral domain is going to be made by calculating the autocovariance functions of both time series and also the cross covariance function of the process at one certain lag k. Then, based on the autocovariance functions $\gamma(k)$ and $\Delta(k)$, and on the cross covariance function of the process $\gamma_{xy}(k)$ one can calculate the (power) spectral density functions for each of the both coordinates and the cross spectral density function (cross spectrum) defined by the relationships below:

$$\begin{cases} f(\omega) = \dfrac{1}{\pi}\sum_{-\infty}^{\infty} \gamma(k)e^{-i\omega} \\ g(\omega) = \dfrac{1}{\pi}\sum_{-\infty}^{\infty} \Delta(k)e^{-i\omega k} \\ f_{xy}(\omega) = \dfrac{1}{\pi}\sum_{-\infty}^{\infty} \gamma_{xy}(k)e^{-i\omega k} \end{cases}$$

CONCLUSIONS

The facts and results in the second chapter of this paper are assuming that the coordinates of the fractal's shape are not correlated one with the other. This case cannot be associated with the process of morphogenesis, because the process of morphogenesis is assuming acorrelation between the two coordinates, caused by subtle interactions between the physical quantities generating the respective coordinates.